Throwing bridges: where and how can classical and quantum views be connected?

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May 21, 2019

Abstract

We have proposed in several recent papers a critical view of some parts of quantum mechanics (QM) that is methodologically unusual because it rests on analysing the language of QM by using some elementary but fundamental tools of mathematical logic. Our approach proves that some widespread beliefs about QM can be questioned and establishes new links with a classical view, which is significant in the debate on the interpretations of QM. We propose here a brief survey of our results, highlighting their common background. We firstly show how quantum logic (QL) can be embedded into classical logic (CL) if the embedding is required to preserve the logical order and not the algebraic structure, and also how QL can be interpreted as a pragmatic sublanguage within a pragmatic extension of CL. Both these results challenge the thesis that CL and QL formalize the properties of different and incompatible notions of truth. We then show that quantum probability admits an epistemic interpretation if contextuality is taken into account as a basic constituent of the language of QM, which overcomes the interpretation of quantum probability as ontic. Finally, we show that the proofs that QM is a contextual theory stand on a supplementary epistemological assumption that is usually unnoticed and left implicit. Dropping such assumption opens the way, at least in principle, to non-contextual interpretations of QM.

Keywords Quantum logic·Quantum pragmatics·Quantum probability·Noncontextual·interpretations of quantum mechanics

1 Introduction

After almost one century after the birth of quantum mechanics (QM), the debate on the interpretation of this theory is still alive (see, e.g., Schlosshauer et al., 2013). At variance with classical mechanics (CM), QM exhibits indeed some features that make it hard to construct a consistent picture of the physical world. In particular, a highly counterintuitive feature of QM is *nonlocality*, following (according to the standard interpretation of QM) from the existence of entangled states, which nevertheless are considered as important resources in quantum information theory. Based on a critical analysis of the theorems aiming at proving nonlocality, we proposed in some previous papers (see, e.g., Garola, 2015; Garola et al., 2016) a falsifiable generalization of QM that embodies the mathematical formalism of QM but introduces a reinterpretation of quantum probabilities able to avoid nonlocality if some restrictive conditions are fulfilled. However, we will not insist on this proposal here, nor consider the numerous interpretations of QM that have been advanced and their variants. We would rather focus on the results obtained in some further papers, in which we made a critical analysis of some typical issues in QM (Garola, 2017; Garola, 2018; Garola and Persano, 2014; Garola and Sozzo, 2010, 2013). Indeed, those results are significant, in our opinion, if one wants to choose or to construct an interpretation of QM, for they show that some widespread beliefs about QM can be questioned.

Let us list the issues that are considered in the papers quoted above.

(i) The *contextuality* of QM, which implies that the values of the observables of a physical system do not pre-exist to their measurements. This is a basic notion in the standard interpretation of QM, but a formal proof of it has been given for the first time by the famous Bell's (1966) and Kochen-Specker's (1967) theorems. It can be rephrased by saying that the properties of a quantum physical system are non-objective, in the sense that assuming that they are either possessed or not possessed by the system independently of any measurement leads to contradictions.

(ii) The controversial role of *(standard, sharp) quantum logic* (QL) in QM. Some authors maintain indeed that QL formalizes a new way of reasoning introduced by QM, as this theory would determine a notion of quantum truth, different and incompatible with the classical notion of truth (see, e.g., Rédei, 1998; Dalla Chiara et al., 2004). Other authors uphold instead that QL is just a part of the mathematical apparatus of QM and has nothing to do with logic (see, e.g., Aerts, 1999).

(iii) The interpretation of *quantum probability*, whose mathematical structure is different from that of classical probability. Indeed, quantum probability is usually maintained to be non-epistemic, in the sense that it cannot be interpreted as expressing an (indirect) measure of our ignorance of values of observables pre-existing to measurements.

Issue (ii) is discussed in (Garola, 2008; Garola, 2017; Garola and Sozzo, 2014) and issue (iii) in (Garola, 2018). Both treatments accept the standard view about non-objectivity of properties. Nevertheless, we obtain some results that question standard beliefs, as anticipated above. Indeed, when considering QL in (Garola, 2008; Garola and Sozzo, 2013), we show that it can be embedded into classical logic (CL) if the embedding is required to preserve the logical order and not the algebraic structure; moreover, we prove in (Garola, 2017) that QL can also be interpreted as a pragmatic sublanguage within a pragmatic extension of CL. Both these results challenge the thesis that CL and QL formalize the properties of different and incompatible notions of truth. Furthermore, when considering quantum probability in (Garola, 2018), we show that it admits an epistemic interpretation if contextuality is taken into account as a basic constituent of the language of QM, which overcomes the aforesaid non-epistemic interpretation of quantum probability.

Finally, issue (i) is discussed in (Garola and Sozzo, 2010; Garola and Persano, 2014). Here we show that the proofs that QM is a contextual theory stand on a supplementary epistemological assumption that is usually unnoticed and left implicit. Dropping such assumption opens the way, at least in principle, to non-contextual interpretations of QM, which are usually excluded on the basis of the conviction that contextuality is "mathematically proven" by Bell's and Kochen-Specker's theorems.

In this paper we present the above arguments in a unified perspective, avoiding technical details but trying to make clear the physical, logical and epistemological reasons underlying them. In particular, we aim to show that our procedures exemplify a methodology that is somewhat unusual in physics, but typical of analytic philosophy. Indeed, they are based on analysing the language of QM by means of some elementary but fundamental tools of mathematical logic, which also leads to establish some new links with classical views. To this end, we anticipate some epistemological and linguistic preliminaries in Section 2 and then summarize the main lines of our analysis of the issues listed above in Section 3 (issue (ii)), Section 4 (issue (iii)) and Section 5 (issue (i); in this case our treatment is also refined with respect to its original presentations), insisting on their common background. We hope that this presentation may constitute a useful contribution to the debate on the possible interpretations of QM.

2 Epistemological and linguistic preliminaries

As we have anticipated in Section 1, we recollect and summarize in this section some notions that are well known to epistemologists (Section 2.1) or to physicists (Sections 2.2 and 2.3). Our presentation, however, is somewhat original, for it mainly focuses on the languages of the theories that are considered (CM and QM), thus constituting a background for our arguments in Sections 3, 4 and 5.

2.1 The received view

According to the *standard epistemological conception*, or *received view* (see, e.g., Braithwaite, 1953; Nagel, 1961; Hempel, 1965; Carnap, 1966), a fully-developed physical theory $\mathcal{T}$ is in principle expressible by means of a metalanguage in which a *theoretical language* L_T and an *observational language* L_O can be distinguished. The theoretical apparatus of $\mathcal{T}$ includes a *mathematical structure* expressed by means of L_T and, usually, an *intended interpretation*, namely a *direct* and *complete* physical model of the mathematical structure. Such an interpretation is often anticipated by the choice of the nouns of the theoretical terms and it is not indispensable in principle, but plays a fundamental role in the intuitive comprehension, justification and development of the theory (think, e.g., to the trajectories of point-like particles in CM or to the interpretation of electromagnetic fields as waves). The observational language L_O, instead, is interpreted via *assignment rules* on an empirical domain, hence it has a semantic interpretation. The two languages are connected by *correspondence* (or *epistemic*) *rules* R_C that establish complex and sometimes problematic relations between L_O and L_T (e.g., a self-adjoint operator may represent many different but physically equivalent measuring devices in QM), so that the correspondence rules together with the assignment rules provide an *empirical interpretation* of the mathematical structure. This interpretation generally is *indirect*, in the sense that there are theoretical entities that are connected with the empirical domain only via *derived* theoretical entities, and *incomplete*, in the sense that only limited ranges of values of the theoretical entities are interpreted (e.g., self-adjoint operators correspond in QM to measuring apparatuses whose outcomes match the eigenvalues of the operators only in finite intervals of the real axis).

The received view was criticized by some authors (see, e.g., Kuhn, 1962; Feyerabend, 1975) and is nowadays maintained to be outdated by several scholars. Nevertheless, we deem that its basic ideas are still epistemologically relevant and may greatly help to single out the fundamental differences between CM and QM. In particular, this view led us to focus our attention on the languages of physical theories, suggesting to explore their similarities and differences by analysing their syntax and semantics. We review in the following sections several results that we have obtained following that suggestion, some of which are quite unexpected and challenge well established beliefs.

We add that in the standard language of physical theories the distinctions introduced by the received view are usually overlooked, and the various linguistic components are mixed together (e.g., the term "observable" may denote in QM a self-adjoint operator on a Hilbert space $\mathcal{H}$, and in this sense it belongs to L_T, but also a physical entity associated with a set of measurement procedures, and in this sense it belongs to L_O; the term "state" may denote a vector of $\mathcal{H}$, but also a physical entity associated with a set of preparing procedures; etc.). Only a rational reconstruction of the language of a theory can lead to distinguish clearly the various elements that occur in it according to the received view. We, however, will deal with this issue only partially in this paper. It will be indeed sufficient for our aims to formalize some simple sublanguages of the languages of CM and QM which exhibit strong similarities and differences.

2.2 The language of classical mechanics

In our (partial) rational reconstruction of the standard language of CM the notions of *material body* and *physical quantity* play a basic role, both at a theoretical and at an observational level. According to the received view (Section 2.1), the terms "material body" and "physical quantity" belong to the observational language L_O of CM. The former refers to a set of elements (*material bodies*) of the empirical domain that are represented in the theoretical language

L_T of CM by the elements of an abstract set, still called material bodies by abuse of language. The latter refers to a set of entities (*physical quantities*) of the empirical domain, each of which consists of a set of (exact) *measurement procedures* and is represented in L_T by a function that takes values on material bodies, still called physical quantity by abuse of language; then, every measurement procedure belonging to a physical quantity, when activated on a material body, performs a *measurement* whose outcome yields the value of the physical quantity on the body.

Based on the notions mentioned above, the derived notions of *property* and *state* can be introduced. Also the terms "property" and "state"belong to L_O. The former refers to a set of entities (*properties*) of the empirical domain, each of which consists of a set of (exact) *dichotomic* measurement procedures and is represented in L_T by a pair of the form (A, Δ), still called property by abuse of language, where A is a physical quantity and $\Delta \in \mathcal{B}(R)$ is a Borel subset of the real line (hence the physical quantity A is bijectively associated with the family $\{(A, \Delta)\}_{\Delta \in \mathcal{B}(R)}$ of properties); then, one says that a material body *a possesses* the property E represented by (A, Δ) if and only if (*iff* in the following) the value that A takes on a belongs to Δ, so that every dichotomic measurement procedure associated, via L_O, with E, when activated on a material body a, performs a measurement whose outcome tells us whether a possesses E. The latter term refers to a set of entities (*states*) of the empirical domain, each of which consists of a set of *preparation procedures* and is represented in L_T by a set of properties, still called state by abuse of language; then, every preparation procedure belonging to a state S, when activated, prepares material bodies sharing the set of properties representing S.

States and properties play a fundamental role in our analysis. Indeed, the statement that a material body a has been prepared by a preparation procedure associated with the state S (briefly, "a is in the state S") can be formalized by an elementary sentence $S(a)$ of predicate logic. Analogously, a statement of the form "a possesses the property E" can be formalized by $E(a)$.

The semantic rules assigning truth values to the sentences of L_O are now crucial. Indeed, truth assignments are made in the language of CM according to the classical theory of truth as correspondence, as reconstructed by Tarski (1944, 1956). When considering elementary sentences, truth values (*true/false*) are assigned by fulfilling Tarski's truth condition (exemplified by the famous statement "'the snow is white' is true iff the snow is white"). This feature of the truth assignment is apparent if one refers to the standard geometrical model of the mathematical apparatus, in which a physical system is represented by a phase space and properties and states by subsets and points, respectively, of the phase space. Indeed, according to such model, a property E is possessed by the individual object a in the state S, that is, the sentence $E(a)$ is true, iff the point representing S belongs to the subset representing E. This rule, however, is not semantically neutral, as such an assignment implies an assumption of *objectivity of properties*, which can be stated as follows.

O. Any property of a material body a is possessed or not possessed by a independently of any measurement.

Usually, assumption O is not stated explicitly but is implicit in the language of CM. It implies in particular that the measurement procedures associated with E, when performed on the empirical object a, check (or reveal, if unknown) the truth value of $E(a)$, which does not depend on the procedures themselves and pre-exists to them: hence, all procedures must yield the same result.

When considering complex sentences that can be formalized by introducing classical logical connectives as $\neg$ (*not*), $\wedge$ (*and*), $\vee$ (*or*), etc., and then connecting elementary sentences of the form $E(a)$ or $S(a)$ by means of these connectives, Tarski's theory requires that truth values be assigned by recursive rules such that the truth value of any complex formula depends only on the truth values of its elementary subformulas: i.e., every truth assignment is *T-functional* (which implies that the meaning of the logical connectives is independent of the empirical interpretation of L_O).

It is now important to observe that assumption O is coupled in CM with an assumption of *compatibility of properties*, which can be stated as follows.

C. The measurement procedures associated with different properties can be performed conjointly.

Assumption C implies indeed that the truth values of the complex sentences of the kind considered above can be checked by checking the truth values of all elementary sentences that occur in them and by using standard truth rules in classical predicate logic. In particular, a sentence of the form $S(a)$ can be seen as logically equivalent to a conjunction of elementary sentences, each of which states that a possesses a given property. Hence, also the truth value of $S(a)$ can be checked (or revealed) by means of measurements

2.3 The language of quantum mechanics

It is well known that QM has been considered a problematic theory since its birth, and that many "interpretations" of it have been proposed. We avoid dealing with this issue here, and refer only to the "standard interpretation" (also "Copenhagen interpretation") of QM, maintaining that QM deals with individual examples of quantum physical systems (briefly, *individual*, or *physical*, *objects*) and their properties (we remind that this option is classified as *realistic* by some scholars; see, e.g., Busch et al., 1996).

When considering the language of the standard interpretation of QM and comparing it with the language of CM, it is apparent that the basic notions are similar, while their relations and interpretations are deeply different. To be precise, the notions of state and property can be defined by replacing the notions of material body and physical quantity with the notions of individual object and observable, respectively, in the definitions introduced in the language of CM. The mathematical apparatus of QM is obviously very different from the mathematical apparatus of CM, but sentences as "the individual object a is in the state S" or "the individual object a possesses the property E" still occur in the language of QM and can be formalized by $S(a)$ and $E(a)$, respectively. Hence classical connectives as $\neg$, $\wedge$, $\vee$, etc., can be formally introduced to construct complex sentences also in QM. Nevertheless, the semantic rules of classical logic are not adequate to match the empirical domain that the language of QM aims to describe. It is well known indeed that in Bohr's holistic view (see, e.g., Bohr, 1958) or in Heisenberg's distinction between "potential" and "actual" properties (see, e.g., Heisenberg, 1958), assumption O in Section 2.2 (objectivity of properties) does not hold, as the outcome of any measurement of a given property on a given individual object depends on the choice of the (macroscopic) measurement procedure. This epistemological view, that has of course an enormous impact on our conception of the physical world and has given rise to a huge literature, is maintained to be "mathematically proven" in QM because of several "no-go" theorems, the most important of which are Bell's (1964, 1966) and Kochen-Specker's (1967). While we think that the proofs of these theorems depend on some implicit epistemological assumptions that may be questioned, thus opening the way to different interpretations of QM (see Section 5), for the moment we maintain the standard view that QM is a *contextual* theory (Bell, 1966; Kochen and Specker, 1967) and that contextuality occurs also at a distance (*nonlocality*: Bell, 1964).

Contextuality and nonlocality have numerous puzzling consequences. We aim to deal with the following in the present paper.

(i) We have seen in Section 2.1 that the intended interpretation of the theoretical language L_T of a physical theory is not logically necessary but plays a fundamental role in the intuitive comprehension of the theory. But contextuality implies, in the case of QM, that it is impossible to supply an intended interpretation (that is, a complete and direct physical model) of L_T in which individual objects are represented together with their properties. Thus, no intuitive picture of the physical world can be given, which may explain why a prominent physicist as Feynmann said "... I think I can safely say that nobody understands quantum mechanics" (Feynmann, 1964). The famous duality between particle and wave models for QM finds its roots in that impossibility.

(ii) When considering a composite quantum system, nonlocality implies that measuring a property of a part of the system may instantaneously actualize a property of another part, even if the latter is far away from the former. This "spooky action at a distance", as Einstein classified it, is commonly accepted nowadays as a consequence of entanglement, but strongly clashes with our intuitive conception of space and time (EPR paradox) even if it does not imply any transmission of information.

(iii) The necessity of giving up assumption O in QM implies that truth values can be assigned to sentences of the form $E(a)$ only by referring to specific measurement procedures. But, then, it turns out that different measurement procedures associated with the same property may yield different outcomes when activated on a given individual object, and that the same measurement procedure, when activated on different individual objects in the same state, may also yield different outcomes. Moreover, it may occur that procedures associated with different properties are incompatible, that is, they cannot be performed conjointly, so that also assumption C in Section 2.2 does not hold in QM. It follows that, even if a formal language with all elementary sentences of the form $S(a)$ and $E(a)$ and connectives $\neg$, $\wedge$, $\vee$, etc., is constructed, no classical semantics can be defined on it in such a way that it formalizes a proper sublanguage of the language of QM. This remark has suggested the idea that a new, non-classical, notion of truth (*quantum truth*) is determined by QM, hence a new logic, i.e. QL. The research on QL started indeed with a famous paper by Birkhoff and von Neumann (1936), which gave rise to an enormous literature, and many scholars maintain that QL formalizes the basic language of QM (see, e.g., Rédei, 1958; Dalla Chiara et al., 2004).

(iv) Classical probability, whose mathematical structure is formalized by Kolmogorov's probability theory, be its "interpretation" logical, or frequentist, or subjectivist, is usually maintained to be *epistemic*, i.e., to express our incomplete knowledge of the empirical world (hence of the truth values of the sentences of the language that describes it). But an elementary sentence of the observational language of QM generally has no truth value before a measurement, hence quantum probability cannot be considered as an (indirect) measure of our ignorance. Therefore it is often classified as *ontic*, and many scholars maintain that it constitutes an intrinsic feature of the physical world. Moreover, quantum probability is defined on the set of *propositions* of QL, which is an orthomodular non-Boolean lattice, hence it does not satisfy Kolmogorov's axioms.

The above consequences of contextuality and nonlocality seem to establish the incompatibility of the classical view of the physical world with the quantum view. As anticipated in Section 1, however, a deeper analysis shows that the classical view can be extended in different directions, which allows to establish some unexpected connections with QL and quantum probability (Sections 3 and 4, respectively). Moreover, we intend to show that, at least in principle, a new interpretation of QM recovering O cannot be excluded (Section 5).

3 Bridging classical and quantum logic

We have seen in Section 2.3 that some authors maintain that QM implicitly determines a new notion of truth, hence a new logic (QL), which is incompatible with CL. Other authors (see, e.g., Aerts, 1999) uphold instead that QL simply formalizes empirical relations among properties in QM, not a new logic.

We have shown in several previous papers that an order structure isomorphic to QL can be singled out inside CL (Garola, 2008; Garola and Sozzo, 2013), and also that CL can be pragmatically extended in such a way that QL can be seen as a part of that pragmatic extension (Garola, 2017). Both procedures establish bridges between the two logics. The former allows a formal embedding of QL into CL preserving a new order (physical preorder) that is implied by the standard logical order but is generally weaker than it (it is well known instead that no embedding of QL into CL preserving the algebraic structure is possible because the algebraic structures of the two logics are different). The latter is innovative and requires a generalization of the pragmatic language introduced by ourselves together with another author several years ago to supply a pragmatic interpretation of intuitionistic logic (Dalla Pozza and Garola, 1995). The two procedures have different merits but can be interconnected by showing that they are based on the same perspective. We resume both of them here focusing mainly on their general features.

3.1 Embedding quantum logic into classical logic

We will present our embedding by referring to the more recent paper on this issue (Garola and Sozzo, 2013). Our starting point is the construction of a "concrete logic", proceeding as follows.

(i) Let $\mathcal{T}$ be a physical theory in which the notions of physical object, property and state are introduced. A classical formal language $L(x)$ is constructed that is intended to express basic relations in $\mathcal{T}$ (hence $L(x)$ is a sublanguage of the theoretical language of $\mathcal{T}$). The syntax of $L(x)$ consists of two parts: firstly, a logical vocabulary, or *alphabet*, which contains two disjoint sets of monadic predicates (called the set $\mathcal{E}$ of *properties* and the set $\mathcal{S}$ of *states*, see Section 2.2), standard connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, an individual variable x and parentheses; secondly, *standard formation rules*, which define a set $\psi(x)$ of elementary and complex *well-formed formulas* (*wffs*). The (formal) semantics of $L(x)$ consists of a *universe* $\mathcal{U}$ of *physical objects*, a set Σ of *interpretations of the variable* x, and, for every $\sigma \in \Sigma$, a *truth assignment* ν_σ that associates a truth value (t/f, where t stands for *true* and f for *false*) with every wff of $\psi(x)$, following classical truth rules. The logical preorder $<$ and the logical equivalence $\equiv$ are then defined on $\psi(x)$ in a standard way, i.e., by setting, for every $\alpha(x)$, $\beta(x) \in \psi(x)$, $\alpha(x) < \beta(x)$ iff, for every $\sigma \in \Sigma$, $\nu_\sigma(\beta(x)) = t$ whenever $\nu_\sigma(\alpha(x)) = t$, and $\alpha(x) \equiv \beta(x)$ iff $\alpha(x) < \beta(x)$ and $\beta(x) < \alpha(x)$.

(ii) A subset $\phi(x) \subset \psi(x)$ is introduced whose elements are all wffs of $\psi(x)$ in which no symbol of state occurs. Based on the classical notion of truth, a derived notion of C-truth is defined on $\phi(x)$ by stating that a wff $\alpha(x) \in \phi(x)$ is *certainly true* (*certainly false*) in a state S iff, for every $\sigma \in \Sigma$, $\nu_\sigma(S(x)) = t$ implies $\nu_\sigma(\alpha(x)) = t$ ($\nu_\sigma(\alpha(x)) = f$).

(iii) A *physical preorder* $\prec$ is defined on $\phi(x)$ by setting, for every $\alpha(x)$, $\beta(x) \in \phi(x)$, $\alpha(x) \prec \beta(x)$ iff, for every $S \in \mathcal{S}$, $\alpha(x)$ certainly true in S implies $\beta(x)$ certainly true in S. Moreover, a *physical equivalence* $\approx$ is defined on $\phi(x)$ by setting, for every $\alpha(x)$, $\beta(x) \in \phi(x)$, $\alpha(x) \approx \beta(x)$ iff $\alpha(x) \prec \beta(x)$ and $\beta(x) \prec \alpha(x)$. Then, it can be proved that $<$ implies $\prec$ and $\equiv$ implies $\approx$.

(iv) A notion of *verification* is defined in $L(x)$ by considering *verifiable* (according to $\mathcal{T}$) all wffs of $\phi(x)$ that are logically equivalent to elementary wffs of $\phi(x)$.

(v) Let $\phi_V(x)$ be the subset of all verifiable wffs of $\phi(x)$, and let $\mathcal{T}$ induce a *weak orthocomplementation* $^\perp$ on $(\phi_V(x), \prec)$ (i.e., a mapping of $\phi_V(x)$ into itself such that, for every $\alpha(x) \in \phi_V(x)$, $(\alpha(x))^{\perp\perp} \approx \alpha(x)$, and for every $\alpha(x)$, $\beta(x) \in \phi_V(x)$, $\alpha(x) \prec \beta(x)$ implies $(\beta(x))^\perp \prec (\alpha(x))^\perp$). Then, the structure $(\phi_V(x), \prec, ^\perp)$ is the *concrete logic*[1] associated with $\mathcal{T}$.

When considering CM, the phase space representation of physical systems shows that C-truth coincides with classical truth on $\phi(x)$. Moreover, all wffs of

[1] We note that the meaning of the term *concrete logic* does not coincide here with the meaning assigned to that term by other authors, see, e.g., Pták and Pullmanová, (1991).

$\phi(x)$ are verifiable, at least in principle, according to CM, so that one can set $\phi_V(x) = \phi(x)$. Hence, the concrete logic of CM has the structure of a classical logic. It is important to note, however, that there may be physical theories of macroscopic systems in which the testability criteria are restricted, so that $\phi_V(x)$ is a proper subset of $\phi(x)$. In these cases, one obtains concrete logics whose algebraic structure may be very different from the structure of Boolean lattice that characterizes CL. Examples of these logics are provided (without referring to $L(x)$) by the macroscopic systems considered by Aerts to show that quantum logical structures can be obtained in suitably chosen macroscopic domains (see, e.g., Aerts, 1999; Garola and Sozzo, 2013).

Aerts' results are relevant because they falsify the belief that QL characterizes QM, as they show that quantum structures can be obtained also in a classical framework. We attain the same conclusion in a generalized form by considering $L(x)$. Indeed, the classical semantics introduced in $L(x)$ implies that $L(x)$ cannot be seen as a sublanguage of the language of QM because of the reasons expounded in Section 2.3. Nevertheless, one can show that, if $\phi_V(x)$ satisfies suitably chosen axioms, then $(\phi_V(x), \prec, ^{\perp})$ is a structure isomorphic to QL (up to an equivalence relation). This conclusion implies that QL can be embedded into CL and interpreted as a structure formalizing the properties of a notion of *true with certainty* within $\phi_V(x)$ rather than the properties of a notion of quantum truth that is alternative to the classical notion of truth. Such interpretation of QL is also supported by some traditional approaches to quantum physics, as Piron's (1976), in which the orthomodular structure of the set of quantum "propositions" is recovered standing on the notion of *true with certainty.*

3.2 Recovering quantum logic within a pragmatic extension of classical logic

We have discussed with another author in a previous paper (Dalla Pozza and Garola, 1995) how to pragmatically extend classical propositional logic in such a way that intuitionistic propositional logic can be embedded into the pragmatic part of the extension and reinterpreted as the logic formalizing the properties of the metalinguistic notion of constructive logical proof. Such a reinterpretation has a deep philosophical meaning. Indeed, it shows that one can avoid introducing an intuitionistic notion of truth, different and incompatible with classical truth, by adopting the perspective of *global pluralism*, according to which many different logical systems can coexist without being in competition, for they formalize the properties of different metalinguistic notions (Haack, 1974, 1978; Garola, 1992).

Our pragmatic extension is based on a distinction that goes back to Frege (1893) and that has given rise to a huge literature in linguistic and philosophical studies: that is, the distinction between the sentences of a language, which can be true or false, and the *assertions* of a speaker who commits himself to the truth of the sentences he is uttering. Indeed, an assertion has not a truth value, but it can only be *justified* (if a proof exists that the asserted sentence is true) or *unjustified* (if no proof exists that the asserted sentence is true). Bearing in mind this distinction, we firstly introduce a standard classical propositional logic $\mathcal{L}$, whose formulas we call *radical formulas.* Then we construct an extension $\mathcal{L}^P$ of $\mathcal{L}$ by adjoining a new category of *logical-pragmatic signs*, which contains an *assertion sign* $\vdash$ and *pragmatic connectives* N, K, A, C and E, to the alphabet of $\mathcal{L}$. By using this extended vocabulary, new formation rules are introduced which recursively define a set of elementary and complex assertive formulas of $\mathcal{L}^P$. Every elementary assertive formula consists of a radical formula preceded by the assertion sign, and every complex assertive formula consists of elementary assertive formulas connected by pragmatic connectives. Then, *pragmatic rules* are introduced which specify the conditions that must be fulfilled whenever a *pragmatic evaluation function* on the set of all assertive formulas of $\mathcal{L}^P$ is given which assigns a *justification value* (*justified*/*unjustified*) to every assertive formula. Such conditions formalize the general features of an informal notion of proof (which can be specified in several different ways, as empirical proof, classical logical proof, intuitionistic logical proof, etc.), and the justification value of an assertive formula of $\mathcal{L}^P$ depends on the semantic assignment of truth values to the radical subformulas that occur in it. Moreover, pragmatic evaluation functions are generally not *J-functional* (i.e., the justification value of a complex assertive formula does not depend only on the justification values of its assertive subformulas) because of the features of the informal notion of proof, which determine an intuitionistic-like behaviour of the pragmatic connectives.

The language $\mathcal{L}^P$ is original from several points of view: in particular, because of the introduction of pragmatic connectives and pragmatic evaluation functions. It provides a pragmatic extension of classical propositional logic that allows us to embed intuitionistic propositional logic in it, as we have stated above. But it may have also a more general role: indeed it provides a framework in which non-classical logics (and not only intuitionistic logic) can be embedded as fragments of its pragmatic part by suitably specifying the informal notion of proof. Hence it was natural for us to wonder, in particular, whether this procedure could apply to QL.

The idea of recovering QL within $\mathcal{L}^P$, however, meets a serious difficulty from the very beginning. Indeed, we have already seen in the previous sections that the language of QM cannot bear a classical semantics. Hence a generalization of the original language $\mathcal{L}^P$ is needed to implement that idea. We introduce such a generalization in the recent paper mentioned above (Garola, 2017), retaining the syntactic apparatus of $\mathcal{L}^P$ and its pragmatic rules, but assuming that the truth assignments on radical formulas can be *partial*, i.e., such that not all elementary and complex radical formulas have a truth value (but classical truth rules for assigning truth values to complex radical formulas are preserved, so that each truth assignment is T-functional in the sense specified in Section 2.2). This generalization makes our pragmatic language ($\mathcal{L}_G^P$ in the following) suitable for dealing not only with the standard interpretation of QM, but also with other interpretations, as the *modal interpretations* or the *objective interpretation* proposed by ourselves on the basis of our criticism of Bell's and Kochen-Specker's theorems (see Section 5). Hence we can single out a fragment

$\mathcal{L}^P_{GQ}$ of $\mathcal{L}^P_G$ that we call *quantum pragmatic language* and introduce a physical (intended) interpretation of it that specifies the notion of proof as the notion of *empirical proof* in QM. This procedure supplies $\mathcal{L}^P_{GQ}$ with a semantics (which can be partial, depending on the interpretation of QM that has been chosen) and a pragmatics (which does not depend on the choice of the interpretation of QM), in which the pragmatic rules mentioned above are fulfilled. It is then easy to show that $(\psi_{AQ}, \prec)$, where ψ_{AQ} denotes the set of all assertive formulas of $\mathcal{L}^P_{GQ}$ and $\prec$ denotes the preorder induced on ψ_{AQ} by the set of all pragmatic evaluation functions, is isomorphic to QL, considered as an order structure, up to an equivalence relation. This result implies that the connectives of QL can be identified, up to an equivalence relation, with (primitive or derived) pragmatic connectives of $\mathcal{L}^P_{GQ}$. Hence QL can be interpreted as a structure formalizing the properties of the notion of *empirical justification* in QM rather than a notion of quantum truth.

The conclusion above matches the conclusion obtained in Section 3.1 if the notion of verification is replaced by the equivalent notion of empirical justification. Moreover, whenever the standard interpretation of QM is adopted, the truth values (*true*/*false*) assigned to the radical formulas match the values of C-truth assigned in Section 3.1 to the formulas of $\phi(x)$. But in Section 3.1 we have seen that QL is proven to be isomorphic (up to an equivalence relation) to an order structure embedded into a classical predicate logic. In the framework presented in this section, instead, QL is embedded into a pragmatic extension of a classical propositional logic, and the embedding is independent of the interpretation of QM that is adopted. This makes the relation between QM and CL more intuitive. The conclusion that QL does not characterize QM is instead made more evident by the approach in Section 3.1. In both cases, however, a bridge between the classical and the quantum views is thrown.

4 An epistemic interpretation of quantum probability

We have recalled in Section 2.3 that quantum probability is maintained to be radically different from classical probability by many scholars. Indeed, besides having a non-classical mathematical structure, it would not admit an epistemic interpretation. This view, however, can be questioned. We have shown indeed in a recent paper (Garola, 2018) that quantum probability can be recovered, under reasonable assumptions that take into account contextuality, as a derived notion in a classical probabilistic framework. This result throws another bridge between the classical and the quantum views, and implies that quantum probability can be considered epistemic, at variance with the standard view.

Our treatment in the paper mentioned above starts from a simple remark. The basic language of QM considered in Section 2.3, which is strongly influenced by the language of CM, makes no reference to contextuality, which is introduced as a complex theoretical notion that can be expressed only by a higher order language. But contextuality is a fundamental feature of QM, which suggests that it should enter its language from the very beginning as an essential notion associated with the notion of property. Moreover, one can suspect that such a change in the basic language of QM could make it possible to introduce a classical semantics on it, thus avoiding the problems mentioned in Section 2.3.

A hint on how to implement the above suggestions is given by our comments on the language of QM in Section 2.3. We can indeed retain all elementary sentences of the form "the individual object a is in the state S", formalized by $S(a)$, and replace every elementary sentence of the form "the individual object a possesses the property E" with the sentence "the individual object a possesses the property E in the context C" or, briefly, "the individual object a possesses the contextual property E_C". A sentence of this kind could then be formalized by $E_C(a)$ in the new basic language of QM.

Of course, the proposal above requires specifying an empirical interpretation of the context C. At first sight, one could think of C as the macroscopic measurement context determined by a measurement procedure associated with a property E. But it is well known that QM predicts that performing a given measurement on different individual objects in the same state may yield different outcomes. This can be intuitively explained by adopting a picture of the world according to which a microscopic world underlies the macroscopic world of our everyday experience and by noticing that there are two possible sources of randomness for the outcomes of a measurement, as follows.

(i) When an individual object is prepared by activating a preparation procedure associated with a state S (see Sections 2.2 and 2.3), we control only macroscopic variables, not the physical situation at a microscopic level. Thus different individual objects produced by the preparation procedure are not bound to possess the same contextual properties.

(ii) When performing a measurement, many *microscopic contexts* (which can be described, in principle, by QM itself) can be associated with the (macroscopic) measurement procedure that is activated, and different microscopic contexts that we cannot control may affect in different ways the outcome of the measurement.

Remark (ii) suggests that C should denote a microscopic context if we want to assign a truth value to a sentence of the form $E_C(a)$. Bearing in mind this suggestion, our first step in the paper mentioned above is constructing a new language intended to serve as a basic elementary language for theories belonging to a class $\mathbb{T}$ in which the notions of physical (or individual) object, state, property and context play a fundamental role. This language is conceived as a generalization of the language $L(x)$ considered in Section 3.1, hence it will still be called $L(x)$ by abuse of language.

The syntax of $L(x)$ consists of two parts, i.e. an *alphabet* and *standard formation rules*. The former contains two disjoint sets $\mathcal{S}$ and $\mathcal{E}_C$ of monadic predicates, with $\mathcal{S}$ a set of *states* and $\mathcal{E}_C = \{E_C = (E, C) | E \in \mathcal{E}, C \in \mathcal{C}\}$ a set of *contextual properties* that is the Cartesian product of a set $\mathcal{E}$ of *properties*, each of which is associated with a set $\mathcal{M}_E$ of *measurement procedures*, and a set $\mathcal{C}$ of *microscopic contexts* (briefly, μ-contexts). Moreover, the alphabet of $L(x)$

contains standard connectives $\neg$, $\wedge$, $\vee$, an individual variable x and parentheses. The formation rules then define in a standard way the set $\Psi(x)$ of elementary and complex *well-formed formulas* (*wffs*) of $L(x)$.

The (formal) semantics of $L(x)$ consists of a *universe* $\mathcal{U}$ of *individual objects*, a set Σ of *interpretations of the variable* x, and for every $\sigma \in \Sigma$, a *truth assignment* ν_σ that associates a truth value (t/f, where t stands for *true* and f for *false*) with every wff of $\Psi(x)$ following classical truth rules (to be precise, an *extension* $ext(\alpha(x)) \subset \mathcal{U}$ of individual objects is associated with every $\alpha(x) \in \Psi(x)$ in such a way that the set of all extensions, ordered by set inclusion, is a Boolean lattice, and, for every $\sigma \in \Sigma$, $\nu_\sigma(\alpha(x)) = t$ iff $\sigma(x) \in ext(\alpha(x))$). Finally, the *logical preorder* $<$ and the *logical equivalence* $\equiv$ are defined on $\Psi(x)$ in a standard way.

It is now important to observe that the truth assignments on $\Psi(x)$ are theoretical functions whose values generally cannot be checked by means of measurements. Indeed, we cannot control the μ-context underlying a measurement, hence we cannot select a measurement whose outcome yields the truth value of an elementary wff $E_C(x)$ when an interpretation σ of the variable x is given. Moreover, several different contextual properties may occur in a complex wff $\alpha(x) \in \Psi(x)$ which refer to different μ-contexts. Hence different measurement procedures may be required to check the truth value of $\alpha(x)$, which raises the problem of their compatibility.

Our second step in (Garola, 2018) is suggested by remark (i). We construct indeed a *μ-contextual probability structure* on $L(x)$ by firstly introducing a probability measure on the Boolean lattice of all extensions and then defining, for every pair $(\alpha(x), \beta(x)) \in \Psi(x) \times \Psi^+(x)$ (where $\Psi^+(x)$ is the subset of all wffs of $\Psi(x)$ whose extension has a non-zero probability measure), a *μ-contextual conditional probability* $p(\alpha(x)|\beta(x)) \in [0,1]$ of $\alpha(x)$ given $\beta(x)$. Such a structure is basically classical, hence μ-contextual conditional probabilities admit an epistemic interpretation. In other words, they can be considered as indexes of our lack of knowledge of the (classical) truth assignments on $\Psi(x)$. However, these probabilities, as the truth assignments considered above, generally cannot be checked by means of measurements. Indeed, checking a μ-contextual conditional probability would require performing the same measurement on many individual objects, maintaining under control both the preparation procedures at a microscopic level and the μ-contexts underlying the measurement, which is impossible, as noticed in remarks (i) and (ii).

Our third step in (Garola, 2018) is then looking for theoretical entities, empirically interpreted via correspondence rules (see Section 2.1), whose values can be checked. To this end we consider, for every property E, the set $\mathcal{M}_E$ of all measurement procedures associated with E, and, for every $M \in \mathcal{M}_E$, the macroscopic measurement context C_M determined by M and the set $\mathcal{C}_M$ of all μ–contexts underlying C_M. Then, we associate a probability to each μ-context $C \in \mathcal{C}_M$. This framework allows us to introduce a binary relation k of *compatibility* on the set $\mathcal{E}$ of all properties (the properties E and F are said to be *compatible* or, briefly, EkF, iff they share at least one measurement procedure; hence k is reflexive, symmetric, but generally not transitive). Moreover, it allows us to select a subset of *testable* wffs of $\Psi(x)$ (a wff $\alpha(x)$ is said to be testable iff either all properties that occur in it share at least one measurement procedure M and are associated with the same μ–context in $\mathcal{C}_M$ or, by convention, no contextual property occurs in it, as in the case of the elementary wff $S(x)$) and to introduce a notion of *joint testability* on $\Psi(x)$ (the wffs $\alpha(x)$ and $\beta(x)$ are said to be *jointly testable* iff their conjunction is testable). Whenever $\alpha(x)$ and $\beta(x)$ are jointly testable, they share some measurement procedures. Choosing one of them, say M, and assuming that, for every $C \in \mathcal{C}_M$, $\beta(x) \in \Psi^+(x)$ (which will be understood in the following), we introduce the average of the μ-contextual conditional probability of $\alpha(x)$ given $\beta(x)$ over all the μ-contexts in $\mathcal{C}_M$. Whenever this average does not depend on the choice of M, we denote it by $< p(\alpha(x)|\beta(x)) >$ and say that it is the *mean conditional probability* of $\alpha(x)$ given $\beta(x)$.

Mean conditional probability is crucial in our approach. Indeed, we can now focus our attention on the subclass $\mathbb{T}' \subset \mathbb{T}$ of theories in which, for every pair $(\alpha(x), \beta(x)) \in \Psi(x) \times \Psi^+(x)$ of jointly testable wffs, the foregoing condition on the average is fulfilled and a mean conditional probability of $\alpha(x)$ given $\beta(x)$) is defined. Then, in every $\mathcal{T} \in \mathbb{T}'$, we can assume that performing a measurement by activating a measurement procedure M shared by $\alpha(x)$ and $\beta(x)$ on a large number of individual objects tells us the truth values of $\alpha(x)$ and $\beta(x)$ for different interpretations of the variable x, which allows us to evaluate frequencies that we assume to check the mean conditional probability $< p(\alpha(x)|\beta(x)) >$. Intuitively, this assumption can be justified by observing that both sources of randomness pointed out in remarks (i) and (ii) underlie M. We therefore call the foregoing repeated activation of M on different individual objects a *mean probability measurement*.

Our general framework is thus completed. Its relevance and usefulness become apparent when considering special cases. Bearing in mind the role of properties and states in QM, let us consider, in particular, the elementary wffs $E_C(x)$ and $S(x)$. Both these wffs are testable (the latter by convention, as stated above). Moreover, they are obviously jointly testable, and we can assume that $S(x) \in \Psi^+(x)$ (the state S would be irrelevant if $ext(S(x))$ had probability measure zero). Thus, for every $E \in \mathcal{E}$ and $S \in \mathcal{S}$ we can consider the mean conditional probability $< p(E_C(x)|S(x)) >$ of $E_C(x)$ given $S(x)$, which we briefly denote by $P_S(E)$ in the following. Hence, we can define a preorder $\prec$ and an equivalence relation $\approx$ on the set $\mathcal{E}$ of all properties (for every E, $F \in \mathcal{E}$, $E \prec F$ iff, for every $S \in \mathcal{S}$, $P_S(E) < P_S(F)$, and $E \approx F$ iff $E \prec F$ and $F \prec E$). Whenever $(\mathcal{E}, \prec)$ is a lattice with an orthocomplementation $^\perp$ (i.e., a mapping of $\mathcal{E}$ into itself such that, for every $E \in \mathcal{E}$, $E^{\perp\perp} = E$, and for every E, $F \in \mathcal{E}$, $E \prec F$ iff $F^\perp \prec E^\perp$) and the mapping $P_S : E \in \mathcal{E} \rightarrow P_S(E) \in [0, 1]$ satisfies some standard conditions (in particular, the probability of the join of disjoint properties is the sum of the probabilities of the properties), we say that P_S *is a generalized probability measure* on $(\mathcal{E}, \prec, ^\perp)$ and call $P_S(E)$ the *Q-probability* of E given S. If $(\mathcal{E}, \prec, ^\perp)$ is not Boolean, then P_S is a non-classical probability measure, hence we conclude that non-classical probabilities can be obtained as derived notions in our classical probabilistic framework. It is then easy to show that P_S, if it is non-classical, does not allow to define in a canonical way a *conditional Q-probability* of a property E given S and another property F. We are thus led to introduce a non-standard definition of conditional Q-probability by considering successive measurements of E and F (we avoid details here for the sake of brevity).

Let us come to QM. If we assume that QM belongs to the class $\mathbb{T}'$ of theories defined above, we obtain some important achievements.

First of all, the quantum probability of a property E in a state S (Born's rule) can be considered as the specific form that the Q-probability $P_S(E)$ takes in QM. Hence we conclude that the non-classical character of quantum probability can be explained in classical terms by taking into account μ-contexts. In particular, quantum probability can be seen as a derived notion in a classical probabilistic framework, hence it can be given an epistemic rather than an ontic interpretation. To be precise, it can be interpreted as an (indirect) measure of our lack of knowledge of the physical situation at a microscopic level, both referring to preparation and to measurement procedures.

Secondly, the reflexive, symmetric but not transitive binary relation of compatibility on E introduced in QM can be seen as the specific form that the relation k takes in QM, which provides a natural explanation of it in a contextual framework.

Thirdly, the quantum notion of conditional probability can be considered as the specific form that the conditional Q-probability takes in QM, so that its non-standard features (in particular, the violation of the Bayes theorem) is explained in terms of contextuality.

We add that our perspective is supported by some previous research in the literature. Indeed, mean conditional probabilities and mean probability measurements are conceptually similar to the *universal averages* and the *universal measurements*, respectively, introduced by Aerts and Sassoli de Bianchi (2014, 2017). Our recognition that two sources of randomness underlie each measurement procedure also agrees with analogous remarks of these authors.

To close, we note that we showed in (Garola, 2018) that CM can be included in the class $\mathbb{T}'$ as a special theory, in which truth assignments on $\Psi(x)$ do not depend on the μ-contexts and the compatibility relation k is trivial.

5 The contextuality of quantum mechanics: a critical view

We have shown in the previous sections that some bridges can be thrown between the classical and the quantum views even if the standard interpretation of QM is adopted. In particular, we have explicitly accepted in Section 4 the standard view according to which QM is a contextual theory. This view rests nowadays on the theorems quoted in Section 2.3, which are maintained to prove that assumption O in Section 2.3 is untenable in QM, hence to provide an irrefutable support to the thesis, going back to the founders of the theory, that the properties of a quantum physical system are only potential before a measurement (or non-objective, according to the terminology adopted in this paper).

However, the conclusion above may be questioned. Indeed, we have shown in some previous papers (see, e.g., Garola and Sozzo, 2010; Garola and Persano, 2014) that the aforesaid theorems rest on a hidden epistemological assumption. If such assumption is accepted, then the standard view follows at once, but if it is rejected, then new interpretations of QM become possible, at least in principle, which avoid contextuality, hence the non-objectivity of properties. We supply in this section a brief account of this issue. The reader can find more complete treatment and bibliography in the papers quoted above.

Let us consider Bell's (1966) and Kochen-Specker's (1967) theorems. These theorems aim to show that (local) hidden variables that would determine the values of all observables of a quantum physical system independently of any measurement cannot exist (which amounts to say, from our present point of view, that assumption O in Section 3.2 is untenable when dealing with quantum physical systems), and many different proofs of them besides the original ones have been provided. But an analysis of such proofs shows that they rest, as stated above, on an epistemological assumption that is usually left implicit. To make the point clear, let us consider a typical scheme of proof of contextuality (see also Mermin, 1993).

(i) A theoretical law Λ of QM is considered (e.g., the conservation of total angular momentum in the case of a spin 1 particle).

(ii) Some laws Λ_1, Λ_2, ... are deduced from Λ, each of which contains only compatible observables and establishes correlations among the possible values of the observables that occur in it (via a *Kochen-Specker condition* that we do not report here for the sake of brevity). But the choice of Λ_1, Λ_2, ... is done in such a way that there are observables in some laws that are not compatible with observables that occur in other laws.

(iii) It is assumed (ad absurdum) that the values of the observables are independent of the measurements that one can perform to check (the predictions of) Λ_1, Λ_2, ...

(iv) It is shown that, if Λ_1, Λ_2, ... are suitably chosen, every assignment of values to all observables that occur in Λ_1, Λ_2, ... contradicts some of the correlations among possible values established by these laws.

(v) The contradiction in (iv) follows from the assumption in (iii), which is therefore untenable (hence the contextuality of QM).

Based on the conclusion in (v), the accepted doctrine states that, whenever we perform a measurement intended to check one of the laws Λ_1, Λ_2, ..., say Λ_i, we determine a context that actualizes some possible values of the (compatible) observables that occur in Λ_i. These values are correlated in such a way that Λ_i is satisfied. But if we check another law, say Λ_j, in which an observable A occurs that also occurs in Λ_i, the measurement context associated with Λ_j may actualize a value of A that is different from the value actualized by the measurement checking Λ_i.

To analyse critically the arguments above, let us refer to the received view summarized in Section 2.1. According to this view, the laws Λ_1, Λ_2, ... can be considered as statements of the theoretical language of QM, but Λ_1, Λ_2, $\ldots$ correspond, via correspondence rules, to laws expressed by statements of the observational language of QM, which we still denote by Λ_1, Λ_2, $\ldots$ by abuse of language (hence Λ_1, Λ_2, $\ldots$ will be considered both as sentences of the theoretical and of the observational language). These laws are *empirical laws*, in the sense that each of them can be checked by means of measurements that establish the values of the observables that occur in it. Every set of measurements checking the law Λ_i determines a (macroscopic) *measurement context.* Thus we can associate with Λ_i a set of measurement contexts (there are generally many ways of checking Λ_i), and QM implies that Λ_i is a true sentence in each of these measurement contexts, so that the correlations among the values of the observables established by Λ_i (see (ii) above) must be fulfilled by the outcomes of the measurements when Λ_i is checked. But the formalism of QM does not imply by itself that Λ_i is true in those physical contexts in which it cannot be checked (e.g., in a measurement context associated with a law $\Lambda_j \neq \Lambda_i$). Stating that Λ_i is true in every physical context is a metalinguistic (epistemological) assumption on the laws of QM (*assumption R* in the following) that can be added to the mathematical formulation of QM but is not implied by it.

Now, reaching the conclusion in (v) requires introducing assumption R besides QM. Indeed, one needs to assume that Λ_1, Λ_2, $\ldots$ are true in every physical context to establish that one cannot accept the contradiction in (iv) and must give up the assumption in (iii). Equivalently, one needs to assume that the conjunction of Λ_1, Λ_2, $\ldots$ must be always true, even if no measurement context can be associated with it because non-compatible observables occur in it.

The introduction of assumption R in the reasoning above, however, usually remains implicit, and most scholars are not aware of it, so that contextuality is maintained to be a mere mathematical consequence of the formalism of QM. Of course, introducing such assumption is not wrong, and no criticism can be done in this sense to the standard view. But it should be considered as an (epistemological) choice, not a logical necessity. Moreover, it should be noted that this choice is consistent with a classical realistic conception of theoretical laws, even if its consequences are highly non-classical when dealing with quantum physical systems because it implies abandoning every attempt at introducing a classical semantics in the language of QM.

One is thus led to inquire the alternative nonstandard choice of dispensing with assumption R. One then concludes that, whenever that choice is made, the conjunction of Λ_1, Λ_2, $\ldots$, if considered as a sentence of the observational language, is not bound to be always true. It joins indeed the empirical laws Λ_1, Λ_2, $\ldots$, each of which is true in its measurement contexts but may be false in physical contexts in which it cannot be checked. All predictions of QM are thus preserved, but the contradiction in (iv) does not imply that the assumption in (iii) must be rejected. Hence the possibility of adopting a classical semantics for the language of QM avoiding contextuality cannot be excluded.

The above arguments suggest a generalization. Indeed, assumption R can be seen as a special case of a general principle (which we called *meta-theoretical classical principle*, or *MCP*, in some previous papers, see, e.g., Garola and Persano, 2014) that states that all physical laws expressed as sentences of the observational language of a physical theory must be true in every physical context. Then we call accepting MCP *standard epistemological position* in the following.

Dispensing with R suggests instead a new general principle (which we called *meta-theoretical generalized principle*, or *MGP*, see again Garola and Persano, 2014) that states that all physical laws expressed as sentences of the observational language of a physical theory are bound to be true in every physical context in which they can be checked, at least in principle, but can be true as well as false in those physical contexts in which the theory itself prohibits that they can be checked (hence different laws may have different *domains of validity*). Then, we call accepting MGP *nonstandard epistemological position* in the following. This position is consistent with interpreting the mathematical apparatus of QM as a calculus whose role is producing laws of the observational language by deduction and by using the correspondence rules. Whenever one of these laws relates only compatible observables, some measurement contexts exist in which the law is bound to be true (i.e., the assignments of values to the observables that occur in it are bound to satisfy the law itself) and in these contexts it can be checked. But if the law relates incompatible observables, then it cannot be checked in any physical context, so that no measurement context exists in which the law is bound to be true (i.e., the assignments of values to the observables that occur in it are not bound to satisfy the law itself). Any realistic conception of the theoretical laws of QM must thus be excluded, which makes the foregoing nonstandard epistemological position more consistent with the "anti-metaphysical" attitude of QM than the standard epistemological position.

Summing up, the standard view adopts a standard epistemological position, hence accepts a widespread conception of the laws of physics, but is then prevented from introducing a classical semantics for the language of QM because of the non-objectivity of properties following from contextuality. An alternative view adopting the nonstandard epistemological position (which, even if nonstandard, is consistent with the received view, see Section 2.1) does not meet with a similar prohibition, which opens the way to alternative interpretations of QM that avoid non-objectivity of properties, thus allowing the introduction of a classical semantics for the language of QM. A new bridge is thus thrown between the classical and the quantum views of the world.

Both the standard and the nonstandard epistemological positions meet, however, some problems. The former implies, in particular, *contextuality at a distance*, or *nonlocality*,[2] which is counterintuitive. The latter may avoid this kind of contextuality but implies a kind of contextuality of empirical physical laws that can be seen as a consequence of the limits of our theoretical knowledge but

[2]Indeed, the proofs nof nonlocality of QM that do not resort to probabilities are given resting an assumption R and following the same scheme described above (see, e.g., Greenberger et al., 1990; Mermin, 1993). The original argument intended to prove nonlocality of QM (Bell 1964), however, was based on probabilities (Bell's inequalities) and requires a specific treatment. We do not deal with this issue here for the sake of brevity (see, e.g., Garola and Pykacz, 2004).

is somewhat uneasy. However, the nonstandard epistemological position has the merit of leaving open the possibility of finding a more general theoretical framework, embedding QM, in which properties are objective. In this framework QM would be regarded as an incomplete theory, recovering a perspective going back to Einstein (1935).

To close, we remind that we have tried in several papers to take some steps towards the construction of the theoretical framework envisaged above (see, e.g., Garola, 2015; Garola et al., 2016). Our proposal (which we called *ESR model*) assumes that any property that a microscopic physical system may possess is either possessed or not possessed by the system (here called *physical object*, as in the previous sections) independently of any measurement, and tries to explain the laws of QM in terms of more general laws. The basic idea consists in assigning a probability that a physical object may remain undetected when a measurement on it is performed because of the set of properties possessed by the physical object itself rather than because of a lack of efficiency of the measurements. The quantum probability that a physical object in a state S turns out to possess the property E when a measurement of E is performed on it would then refer to the set of physical objects that can be detected in this kind of measurement rather than to the set of physical objects that are prepared in the state S. Thus, our ESR model modifies the standard interpretation of quantum probabilities but preserves the mathematical apparatus of QM, which is embedded into a more general mathematical framework. Moreover, the ESR model predicts that there are upper limits (depending on the measurements that are considered) to the percentage of objects that can be detected, which makes it falsifiable.

Acknowledgement. The author is greatly indebted with Dr. Antonio Negro for carefully reading the manuscript and providing many useful remarks and suggestions.

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